Stirling engine regenerators: How to attain over 95% regenerator effectiveness with sub-regenerators and thermal mass ratios

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HIGHLIGHTS

• Develop heat transfer model to predict temperature in regenerator and working fluid.
• Derive expressions for regenerator effectiveness and Stirling engine efficiency.
• Require minimum of 19 sub-regenerators to attain 95% regenerator effectiveness.
• Uniform sub-regenerator thermal mass ratio distribution maximizes effectiveness.
• Experimentally validate heat transfer model.

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ABSTRACT

A combined theoretical and experimental approach is used to determine how to achieve a desired value for the Stirling engine regenerator effectiveness. A discrete one-dimensional heat transfer model is developed to determine which parameters influence the effectiveness of Stirling engine regenerators and quantify how they influence it. The regenerator thermal mass ratio and number of sub-regenerators were found to be the two parameters that influence the regenerator effectiveness, and the use of multiple sub-regenerators is shown to produce a linear temperature distribution within a regenerator, which enables the effectiveness to be increased above 50%. It is shown that increasing the regenerator thermal mass ratio and number of sub-regenerators results in an increase in regenerator effectiveness and a corresponding increase in the Stirling engine efficiency. A minimum of 19 sub-regenerators are required to attain a regenerator effectiveness of 95%. Experiments validated the heat transfer model, and demonstrated that stacking sub-regenerators, such as wire meshes, provides sufficient thermal resistance to generate a temperature distribution throughout the regenerator. This is the first study to determine how Stirling engine designers can attain a desired value for the regenerator effectiveness and/or a desired value for the Stirling engine efficiency by selecting appropriate values of regenerator thermal mass ratio and number of sub-regenerators.

1. Introduction

Stirling engines have a high potential to alleviate some of the increasing global demand for clean energy. This potential is due in part to the fact that Stirling engines are external combustion engines that can therefore be powered by a wide range of heat sources, including sustainable options such as solar thermal, waste heat, and biofuels [1]. Additionally, Stirling engines have the ability to attain the highest theoretical engine efficiency, primarily due to the use of a regenerator, which exchanges thermal energy with the working fluid after each isothermal expansion and compression stroke, thus minimizing the amount of thermal energy the heater and cooler must either add or reject, respectively. Accordingly, regenerator design and development has received much attention in past research, and has been a focal point in the process of enhancing Stirling engine performance and efficiency. Characteristics of an effective regenerator are a high thermal storage and heat transfer capacity, a limited resultant pressure drop, and a sufficient temperature distribution to ensure that the working fluid alternates between target temperatures, $T_H$ and $T_L$. It was recently established by Dai et al. [2] that in order for a regenerator to attain a regenerator effectiveness higher than 50%, it is required that the regenerator be divided into discrete thermally isolated components, referred to as sub-regenerators. Interesting questions that arise in the design of regenerators are: how does the regenerator thermal mass ratio

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and number of sub-regenerators influence the effectiveness of regenerators? How can one design a regenerator to attain 95% effectiveness or higher? Can parallel plates or other continuous regenerators attain higher than 50% effectiveness? Does stacking wire meshes reduce the regenerator to a single sub-regenerator and limit its effectiveness to a maximum of 50%? In order to further improve the design and our understanding of Stirling engine regenerators, a comprehensive investigation to determine the influence of the regenerator thermal mass ratio and number of sub-regenerators on the effectiveness of regenerators is required to answer these questions.

In past literature, significant effort has been invested in quantifying the thermal and flow characteristics of various types of regenerators. Experiments have implemented steady flow conditions to quantify the pressure drop [3] and heat transfer characteristics [4] in wire mesh regenerators, while other experiments have imposed oscillatory flow conditions to measure the pressure drop and heat transfer rate in wire mesh regenerators [5,6], parallel-plate regenerators [7], and sponge metal regenerators [8]. Combined experimental and numerical studies have quantified the pressure drop [9,10] and heat transfer rate [11] in wire mesh regenerators subjected to oscillatory flow conditions. Other experiments have investigated the influence of the geometry [12] and the type of material used in regenerators on the performance of Stirling engines, including copper and stainless steel [13], and aluminum and Monel 400 [14,15]. Numerical studies have developed Nusselt number [16,17] and friction factor [18,19] correlations for wire mesh regenerators, while other numerical studies have developed similar correlations for micro-channel regenerators [20], segmented involute-foil regenerators [21], and circular miniature-channel regenerators using 3D CFD [22,23] and Reynolds-Averaged Navier-Stokes (RANS) modeling [24]. Other numerical studies have examined the entropy generation in wire mesh regenerators [25] and micro-channel regenerators [26]. Past analytical studies have developed heat transfer models for counterflow regenerators in Stirling engine applications to examine the influence of the flush ratio [27,28] and the number of transfer units (NTU) [29] on regenerator effectiveness. Optimization analyses have varied the regenerator’s fill factor [30], wire diameter [31,32], length [33], and mesh size [34] to maximize Stirling engine performance and efficiency. Investigations into the performance of Stirling engines using an isothermal model [35,36] and a finite time thermodynamic model [37] have varied the effectiveness of the regenerator without discussing what parameters would alter the regenerator effectiveness. Many studies have also assumed perfect regeneration when analyzing Stirling engines at maximum power conditions [38,39], when comparing the net work output between Stirling and Ericsson engines [40], and when assessing the efficiency reduction in Stirling engines due to sinusoidal motion [41]. The previous studies have played a vital role in the advancement of Stirling engine technologies and have elucidated much about the effect of regenerators on engine performance, but it is still not understood how the regenerator thermal mass ratio and number of sub-regenerators influence the effectiveness of regenerators, and how to design regenerators to attain a desired effectiveness.

There have been limited studies that have examined the influence of either the number of sub-regenerators or regenerator thermal mass ratio on regenerator effectiveness or Stirling engine performance and efficiency, and no study has investigated them in tandem. Dai et al. [2] theoretically showed that a regenerator must have an unevenly distributed temperature to attain an effectiveness greater than 50%, and that an unevenly distributed temperature can be accomplished using a number of smaller heat reservoirs to divide the regenerator into sub-
regenerators. A study conducted by Jones [42] examined the effects of the regenerator thermal mass ratio on the ineffectiveness of the regenerator and power loss factor of the Stirling engine. The work found that regenerator thermal mass ratios between 20:1 and 40:1 can influence the performance of Stirling engines, and was not concerned with recommending an ideal value for the regenerator thermal mass ratio. Zarinchang et al. [43] examined the pressure drop and enthalpy flux through a wire mesh regenerator and indirectly found that, for their system, the optimum value for the regenerator thermal mass ratio was 14.9:1. The GPU-3 and 4L23 Stirling engines [44,45] had regenerator thermal mass ratios in the range of 205:1–235:1, and the GPU-3 engine used 308 stacked wire mesh sub-regenerators as its regenerator. Such high values of regenerator thermal mass ratios and number of sub-regenerators require a considerable amount of regenerator material, which inhibits the number of sub-regenerators required to attain specific regenerator effectiveness and generates unnecessary pressure drops. These studies demonstrate that the regenerator thermal mass ratio and quantity of sub-regenerators have a significant impact on the effectiveness of regenerators and Stirling engines, but to date, there has not been any research that has determined the regenerator thermal mass ratio and number of sub-regenerators required to attain specific regenerator effectiveness values without employing unnecessarily high values that cause increased pressure drops. These studies demonstrate that the regenerator thermal mass ratio and quantity of sub-regenerators have a significant impact on the effectiveness of regenerators and Stirling engines, but to date, there has not been any research that has determined the regenerator thermal mass ratio and number of sub-regenerators required to attain specific regenerator effectiveness values without employing unnecessarily high values that cause increased pressure drops. The understanding of how to select these values will lead to enhanced regenerator designs capable of high effectiveness and low pressure drops.

In this paper, a discrete one-dimensional transient heat transfer model of a Stirling engine regenerator was developed to track the thermal response of the working fluid and regenerator simultaneously. A parametric analysis of the transient model determined that only the regenerator thermal mass ratio influences the settling temperature of the regenerator and working fluid, and that the remaining transient parameters can be neglected. Accordingly, a heat transfer model was developed to quantify the settling temperature of the regenerator and working fluid, and the remaining transient parameters of the two isochoric (constant volume) regeneration processes of the Stirling cycle. During these processes, energy is exchanged between the working fluid and the Stirling engine’s regenerator, which is shown in Fig. 1, and their temperatures change conversely to one another. This means that an incremental increase of energy in one of the mediums must be balanced by the decrease of energy in the other, which is expressed as:

\[ -m_c c_v (T_f^{p+1} - T_f^p) = m_c c_v (T_r^{p+1} - T_r^p) \]  

where superscript \( p \) denotes the current time step, and \( p + 1 \) denotes the successive step. The rate of energy change must also be balanced, such that the rate of heat transfer in the regenerator is equal to the convection at the surface of the regenerator. We assume local thermal equilibrium within the regenerator material and working fluid, thus neglecting local conduction heat transfer within each medium, resulting in:

\[ \rho V_c \frac{dT_f}{dt} = h_s (T_f - T_r) \]  

For thermal equilibrium to be valid for the working fluid would require sufficient mixing and small length scales, which we expect in Stirling engine regenerators. To ensure that the thermal equilibrium approximation is valid for the solid regenerator material, the following condition must be satisfied:

\[ \frac{hr}{2k_v} < 0.1 \]  

That is, the Biot number must be below the threshold value of 0.1 [46]. The largest expected value of the Biot number for the work analyzed in this paper corresponds to the regenerator material with the

![Fig. 1. Schematic of a Stirling engine with a regenerator comprised of five sub-regenerators, a hot piston, cold piston, heater, and cooler.](image)
lowest thermal conductivity (stainless steel: $k = 16.2 \text{ W/m/K}$), the largest radius ($r = 0.04 \text{ mm}$), and the highest heat transfer coefficient ($h = 18,000 \text{ W/m}^2\text{K}$), which results in a value of 0.022. Since this is below the threshold value, the approximation will be valid.

To model the transient thermal response of the working fluid and regenerator, Eq. (2) was discretized as follows:

$$\rho c_r \left( \frac{T_{p+1} - T_p}{\Delta t} \right) = h A (T_f - T_p)$$

(4)

Assuming that the regenerator is made up of a cylindrical material, where the length is substantially larger than the radius, we can write $V/A = r/2$, and simplify Eq. (4) to:

$$T_{p+1} = \frac{2h\Delta t}{\rho c_r r} T_f + \left( 1 - \frac{2h\Delta t}{\rho c_r r} \right) T_p$$

(5)

Eq. (1) is then rearranged as follows:

$$T_{p+1}^f = T_f^r - \gamma (T_{p+1}^f - T_f^r)$$

(6)

where the regenerator thermal mass ratio, $\gamma$, is:

$$\gamma = \frac{m_r c_r}{m_f c_f}$$

(7)

Eqs. (5) and (6) model the thermal response of the regenerator and working fluid, respectively, and they reveal that the parameters that influence the transient response of the system are the radius of the regenerator, $r$, material properties of the regenerator, $c_r$ and $\rho$, the heat transfer coefficient, $h$, and the regenerator thermal mass ratio, $\gamma$. The value of the time step, $\Delta t$, was set low enough to maintain stability of the simulation and to ensure it did not influence the temperatures. To examine how each of the parameters influence the settling temperature of the regenerator and working fluid, a parametric analysis was conducted.

Fig. 2. Transient response of a single regenerator (one sub-regenerator) absorbing energy from the working fluid for: (a) varying $h$ with constant $r = 0.02 \text{ mm}$, constant regenerator material (stainless steel), and constant $\gamma = 5:1$, (b) varying $r$ with constant $h = 18,000 \text{ W/m}^2\text{K}$, constant regenerator material (stainless steel), and constant $\gamma = 5:1$, (c) varying regenerator material with constant $r = 0.02 \text{ mm}$, constant $h = 18,000 \text{ W/m}^2\text{K}$, and constant $\gamma = 5:1$, and (d) varying $\gamma$, constant $r = 0.02 \text{ mm}$, constant $h = 18,000 \text{ W/m}^2\text{K}$, and constant regenerator material (stainless steel).
undertaken.

2.2. Transient parametric analysis of a single regenerator

A parametric analysis of the transient heat transfer model was conducted for a single regenerator (one sub-regenerator) to reveal how the temperatures of the regenerator and working fluid are impacted by the regenerator radius, heat transfer coefficient, material properties, and regenerator thermal mass ratio. The regenerator radius, material, and heat transfer coefficient were varied in Fig. 2(a)–(c) to assess how they impact the settling temperature of the system, which is the settling temperature of both the regenerator and working fluid. The influence of the regenerator thermal mass ratio, \( \gamma \), was then examined in Fig. 2(d). The values were chosen based on the specifications of a GPU-3 Stirling engine [44,45], which has a stainless steel regenerator with a wire radius of \( r = 0.02 \) mm, and we calculated a heat transfer coefficient of approximately \( h = 18,000 \text{ W/m}^2\text{K} \) for the entire regenerator mesh, using [47]:

\[
\text{Nu} = \frac{\text{hD}}{k_f} = C_2C_1Re^{0.8}Pr_{avg}^{0.5} \left( \frac{P_{avg}}{P_f} \right)^{1.2}
\]  

A regenerator thermal mass ratio of \( \gamma = 5:1 \) was selected for the plots in Fig. 2(a)–(c) to illustrate the influence of these parameters on the regenerator and working fluid settling temperatures, and the value of \( \gamma \) was then varied for Fig. 2(d). The results from a regeneration process of a single sub-regenerator with air moving from hot \((T_i = 1000 \text{ K})\) to cold \((T_i = 300 \text{ K})\) are plotted in Fig. 2(a)–(c), and they reveal that the heat transfer coefficient, radius, and material have an impact on the rapidity of the transient response but have no influence on the settling temperature of an individual cycle. The GPU-3 Stirling engine operates at rotational speeds at or below a value of 3,500 rpm [44,45], which corresponds to a minimum regeneration time of approximately 0.004 s. The settling time in Fig. 2(a)–(c) is a maximum of 0.004 s, which indicates that the regenerator temperature will reach the settling temperature during each regeneration process. Therefore, in these cases, the settling time is irrelevant and these parameters will not have an influence on the effectiveness of the regenerator or performance of the engine. The only way to influence the regenerator’s effectiveness is to change the value of the equilibrium temperature, which can be accomplished by varying the remaining parameter, the regenerator thermal mass ratio.

The response of four different regenerator thermal mass ratios were plotted in Fig. 2(d) for a regeneration process with air moving from hot \((T_i = 1000 \text{ K})\) to cold \((T_i = 300 \text{ K})\), and it is shown that the regenerator thermal mass ratio not only impacts the rapidity of the system response, but also the system settling temperature. The value of the system settling temperature is an important consequence of the regenerator thermal mass ratio since it indicates how much additional thermal energy the heater and cooler of the Stirling engine must either have as a single regenerator and the whole regenerator is then composed of a series of thermally isolated sub-regenerators. This approximation will be validated by experiments and shown to yield accurate temperature predictions for regenerators with multiple sub-regenerators. Having determined that the regenerator thermal mass ratio is the sole parameter that influences the system settling temperature, only Eq. (6) is required to model the settling temperature of regenerators composed of a number of sub-regenerators and the working fluid. Since the settling time has been shown to be irrelevant, Eq. (6) can be rewritten without successive time steps, and the \( p \) superscript is replaced with a subscript denoting the initial temperature (indicating the temperature at the start of each regeneration process), while the \((p + 1)\) superscript is replaced with a subscript denoting the final temperature (indicating the temperature at the end of each regeneration process). We then simplify the heat transfer model by rearranging Eq. (6) for a system settling temperature, which is the condition when the working fluid and each sub-regenerator attain the same temperature:

\[
T_{sys} = T_{f\text{,final}} = T_{i\text{,final}}
\]  

and Eq. (6) becomes:

\[
T_{sys} = \frac{T_{f\text{,initial}} + \gamma T_{i\text{,initial}}}{\gamma + 1}
\]  

Four versions of Eq. (15) are required to model the system settling temperature for a regenerator with multiple sub-regenerators. To model one regenerative cooling process, where the regenerator is absorbing thermal energy from the working fluid, we require the following two

\[
\tau = \frac{\rho c_p r}{2h(1 + \gamma)}
\]  

\[
\tau_{reg} = \frac{15}{\omega}
\]  

Eq. (9) corresponds to the time when the regenerator and working fluid temperatures are within 62.2% of the system settling temperature, and we must therefore multiply the natural time scale by 5 to ensure that they are within 99% of the system settling temperature, hence:

\[
\frac{\tau}{\tau_{reg}} = \frac{5 \rho c_p r}{2h(1 + \gamma)}
\]  

To ascertain that there is sufficient time for regeneration to occur, we evaluate:

\[
\frac{\tau_{reg}}{\tau} < 1
\]  

which results the following expression, denoted as \( \zeta \):

\[
\zeta = \frac{\rho c_p r}{6h(1 + \gamma)} < 1
\]  

Values of \( \zeta \) less than one indicate that more than 99% of the available heat is being exchanged within the given window of time, while values greater than one indicate that the regenerator’s specifications are inadequate to exchange 99% of the available heat with the working fluid. The \( \zeta \) values are labelled in Fig. 2(a)–(d). Stirling engine designers can now evaluate whether or not the regenerator is transferring sufficient thermal energy within the given window of time, and can vary the regenerator’s specifications to ensure that they are maximizing the heat transfer between the working fluid and regenerator with the use of Eq. (13). In summary, Eqs. (3) and (13) are the two conditions that must be satisfied to confirm that only the regenerator thermal mass ratio influences the settling temperature of the system for a single regenerator, and correspondingly the effectiveness of the regenerator. Accordingly, a heat transfer model that considers only the regenerator thermal mass ratio is now developed and expanded to the case of a regenerator composed of multiple sub-regenerators.

2.3. Heat transfer model for multiple sub-regenerators

Having shown that for a single regenerator the thermal mass ratio is the only valid parameter, we now turn our attention to the case of multiple sub-regenerators. To model a regenerator with multiple sub-regenerators we make the assumption that each sub-regenerator behaves as a single regenerator and the whole regenerator is then composed of a series of thermally isolated sub-regenerators. This approximation will be validated by experiments and shown to yield accurate temperature predictions for regenerators with multiple sub-regenerators. Having determined that the regenerator thermal mass ratio is the sole parameter that influences the system settling temperature, only Eq. (6) is required to model the settling temperature of regenerators composed of a number of sub-regenerators and the working fluid. Since the settling time has been shown to be irrelevant, \( \xi \) can be rewritten without successive time steps, and the \( p \) superscript is replaced with a subscript denoting the initial temperature (indicating the temperature at the start of each regeneration process), while the \((p + 1)\) superscript is replaced with a subscript denoting the final temperature (indicating the temperature at the end of each regeneration process). We then simplify the heat transfer model by rearranging Eq. (6) for a system settling temperature, which is the condition when the working fluid and each sub-regenerator attain the same temperature:

\[
T_{sys} = T_{f\text{,final}} = T_{i\text{,final}}
\]  

and Eq. (6) becomes:

\[
T_{sys} = \frac{T_{f\text{,initial}} + \gamma T_{i\text{,initial}}}{\gamma + 1}
\]  

Four versions of Eq. (15) are required to model the system settling temperature for a regenerator with multiple sub-regenerators. To model one regenerative cooling process, where the regenerator is absorbing thermal energy from the working fluid, we require the following two
equations:

\[
T_{\text{sys},H}^{(1)} = \frac{T_H + \gamma^{(1)}T_{\text{sys},L}^{(1)}}{\gamma^{(1)} + 1}
\]

(16)

\[
T_{\text{sys},L}^{(1)} = \frac{T_L + \gamma^{(1)}T_{\text{sys},H}^{(1)}}{\gamma^{(1)} + 1}
\]

(17)

where Eq. (16) is the system settling temperature of the first sub-regenerator for one regenerative cooling process, and Eq. (17) is the system settling temperature of each of the subsequent sub-regenerators, with their order denoted by the superscript \( i \). To model one regenerative heating process, where the regenerator is returning thermal energy to the working fluid, we require the following two equations:

\[
T_{\text{sys},L}^{(N)} = \frac{T_L + \gamma^{(N)}T_{\text{sys},H}^{(N)}}{\gamma^{(N)} + 1}
\]

(18)

\[
T_{\text{sys},H}^{(N)} = \frac{T_H + \gamma^{(N)}T_{\text{sys},L}^{(N)}}{\gamma^{(N)} + 1}
\]

(19)

where Eq. (18) is the system settling temperature of the \( N \)th or last sub-regenerator, and Eq. (19) is the system settling temperature of each of the subsequent sub-regenerators in reverse order since the working fluid is flowing back through the regenerator. The regenerative thermal mass ratio for each sub-regenerator is a fraction of the total regenerator thermal mass ratio, and in the case of a uniformly distributed mass, each value of \( \gamma^{(i)} \) would be equal to \( \gamma/N \). It should be noted that modelling a system with one sub-regenerator requires only Eqs. (16) and (18). The superscript \( i \) is the sub-regenerator location (i.e. second sub-regenerator, third sub-regenerator, etc.) and \( N \) is the total number of sub-regenerators that the regenerator is composed of, where \( 1 < i < N \).

The operation of a Stirling engine is cyclical, so the regenerative processes described by Eqs. (16)–(19) must be repeated over a number of cycles to simulate the thermal behaviour. The Stirling engine and regenerator will reach a steady operating condition after a number of cycles where the system settling temperatures in Eqs. (16)–(19) no longer change. To model the cyclical behaviour for a single regenerator we begin by assuming the working fluid is in the hot side at a temperature of \( T_H \) and the regenerator is initially at a temperature of \( T_{\text{sys},H}^{(1)} = T_L \). We then use Eq. (16) for the first regenerative cooling process (we denote these with \( n \), so for the first one \( n = 1 \)), and substitute the resulting hot system settling temperature (\( T_{\text{sys},H}^{(1)} \)) into Eq. (18) to describe the subsequent regenerative heating process (\( n = 2 \)). The resulting cold system settling temperature (\( T_{\text{sys},L}^{(1)} \)) from Eq. (18) is then substituted into Eq. (16) to model the subsequent regenerative cooling process (\( n = 3 \)). This process is repeated and produces the following summation when rearranged for the hot system settling temperature:

\[
T_{\text{sys},H}^{(1)} = T_H \sum_{n=1}^{\infty} \frac{\gamma^{(n-1)}}{(1 + \gamma)^{2n-1}} + T_L \left( \sum_{n=1}^{\infty} \frac{\gamma^{(n-1)}}{(1 + \gamma)^{2n}} + \frac{\gamma^3}{(1 + \gamma)^3} \right)
\]

(20)

Fig. 3. Schematic diagram of the heat transfer model once steady engine operation has been reached for one complete Stirling cycle. Illustrating the regenerative heating and cooling processes for a regenerator composed of 3 sub-regenerators, \( N = 3 \), and regenerator thermal mass ratio, \( \gamma = 20:1 \).
The regenerator effectiveness is only a function of the target temperatures \( T_{H} \) or \( T_{L} \) and the system settling temperatures \( T_{\text{sys}H} \) or \( T_{\text{sys}L} \), so we can substitute Eq. (21), which is the hot system settling temperature for a single regenerator after steady operating conditions have been reached, to yield the regenerator effectiveness for a single regenerator:

\[
e_{\text{reg}} = \frac{\gamma}{2\gamma + 1}
\]

Similarly, for a regenerator with multiple sub-regenerators, the regenerator effectiveness for steady operating conditions simplifies to:

\[
e_{\text{reg}} = \frac{N\gamma}{(N + 1)\gamma + N}
\]

Since Stirling engines operate primarily in steady operating conditions, with only a short warm-up period, the remainder of the analysis will consider the thermal behaviour once steady operating conditions have been reached.

### 2.4. Stirling engine efficiency model

The Stirling engine efficiency is commonly listed as the Carnot efficiency:

\[
\eta_{\text{Carnot}} = \eta_{\text{Stirling, ideal}} = 1 - \frac{T_{L}}{T_{H}}
\]

however, this efficiency considers ideal regeneration (\( e_{\text{reg}} = 1 \)). The derivations in the previous section revealed that regenerator effectiveness is a function of the regenerator thermal mass ratio and number of sub-regenerators. For non-ideal regeneration, the Stirling engine efficiency must also be a function of the regenerator thermal mass ratio and number of sub-regenerators, and we derive an expression to capture this dependence. The Stirling engine efficiency with non-ideal regeneration is expressed as:

\[
\eta_{\text{Stirling}} = \frac{Q_{\text{in}} - Q_{\text{out}}}{Q_{\text{in}}}
\]

where:

\[
Q_{\text{in}} = m_{f}RT_{H}\ln(\lambda) + Q_{\text{non-reg}}
\]

\[
Q_{\text{out}} = m_{f}RT_{L}\ln(\lambda) + Q_{\text{non-reg}}
\]

and:

\[
\lambda = \frac{V_{\text{max}}}{V_{\text{min}}}
\]

An additional \( Q_{\text{non-reg}} \) term in Eqs. (28) and (29) is required since it represents the amount of supplemental thermal energy that the heater and cooler of the Stirling engine must either add or reject, respectively, to attain target temperatures, \( T_{H} \) and \( T_{L} \), due to non-ideal regeneration. The regeneration process in the Stirling cycle is isochoric, hence the additional heat added and rejected is:

\[
Q_{\text{non-reg}} = (1 - e_{\text{reg}})m_{f}c_{v,f}(T_{H} - T_{L})
\]

Eq. (31) is then substituted into Eqs. (28) and (29), which are then substituted into Eq. (27) to generate an expression for the thermal efficiency of the Stirling engine that incorporates the influence of the regenerator effectiveness (Eq. (25)), as follows:

\[
\eta_{\text{Stirling}} = \frac{R(T_{H} - T_{L})\ln(\lambda)}{RT_{H}\ln(\lambda) + (1 - e_{\text{reg}})c_{v,f}(T_{H} - T_{L})}
\]

It is shown in Eq. (32) that the efficiency of the Stirling engine is not only dependent on the target temperatures, \( T_{H} \) and \( T_{L} \), as seen in Eq. (26), but is also a function of the compression ratio, \( \lambda \), the properties of the employed fluid, \( c_{v,f} \) and \( R \) (specific gas constant), and the regenerator effectiveness, \( e_{\text{reg}} \), which is a function of the regenerator thermal mass ratio, \( \gamma \), and number of sub-regenerators, \( N \), according to Eq. (25).

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**Fig. 4.** Experimental setup of the regenerator test apparatus with (1) linear pneumatic actuators, (2) hot cylinder, (3) regenerator chamber, (4) cold cylinder, (5) PID controller, (6) air-flow regulator, and (7) data acquisition module.
3. Experimental apparatus

We designed and built a regenerator test apparatus to validate the heat transfer model developed in Section 2.3. The regenerator test apparatus, which is shown in Fig. 4, consisted of two linear pneumatic actuators that were programmed to move the hot and cold pistons in an oscillatory manner for a constant volume regeneration process at an equivalent rotational speed of 180 rpm. The apparatus also employed a PID controlled electric heat source, an air-flow regulator to control the motion of the pistons, K-type thermocouples for temperature measurements, and a data acquisition module to process the signal generated by the thermocouples. The thermocouples had an accuracy of ± 1.5 K. Each sub-regenerator consisted of a 304 stainless steel wire mesh with a wire diameter of approximately 0.04 mm, which are the same specifications that were reported for the GPU-3 Stirling engine [44,45]. The two regenerator configurations that were examined in the experiments were spaced and unspaced sub-regenerators. The number of sub-regenerators was set at both 12 and 16 sub-regenerators, which corresponds to a regenerator thermal mass ratio of γ = 4.01:1 and γ = 5.35:1, respectively. An unspaced sub-regenerator configuration notes that there is physical contact between neighbouring sub-regenerators, while a spaced configuration indicates that each sub-regenerator was separated by a ceramic insulation ring to actively minimize the local axial conduction between each sub-regenerator. Thermocouples were placed between every group of four sub-regenerators to record the temperature at the 2nd, 6th, 10th, and 14th sub-regenerators. The heat source and heat sink temperatures were measured using thermocouples that recorded the inside air temperatures of both hot and cold cylinders, and their respective temperatures were maintained at approximately Th = 353 K (80 °C) and Tc = 303 K (30 °C) for every test case. Each experiment was run for at least 40 min before recording measurements to ensure that the temperatures of the working fluid and regenerator reached steady operating conditions. The results gathered from the experiments were then compared against those obtained from the heat transfer model to assess its validity.

4. Results and discussion

4.1. Single regenerator analysis

We first analyze a single regenerator to determine the limitations for regenerator effectiveness of a single regenerator setup. The regenerator effectiveness described in Eq. (24) is only a function of the regenerator thermal mass ratio, and thus is plotted in Fig. 5 against a range of values for the regenerator thermal mass ratio. It can be seen in Fig. 5 that the maximum effectiveness a single regenerator can attain is 50%. As the regenerator thermal mass ratio increases, the regenerator effectiveness increases asymptotically towards a maximum value of 50%. This maximum effectiveness of a single regenerator applies to any regenerator composed of a continuous material with high thermal conductivities and small thicknesses, satisfying the conditions from Eqs. (3) and (13), such as parallel plates. From this analysis it is demonstrated that single regenerators, such as parallel plate regenerators, with specifications that satisfy the thermal equilibrium approximation, cannot attain a regenerator effectiveness greater than 50%. The following sections in this study will reveal how to obtain a regenerator effectiveness greater than 50%, and how to design for an effectiveness greater than 95%.

4.2. Multiple sub-regenerator analysis

4.2.1. Influence of number of sub-regenerators

Using Eqs. (16)–(19), the temperature distributions of regenerators composed of various numbers of sub-regenerators for a regenerative cooling process were plotted in Fig. 6. For this analysis the regenerator thermal mass ratio remained constant and the regenerator mass was distributed evenly across the sub-regenerators. It is shown that as the number of sub-regenerators increases, the temperature of the last sub-regenerator and correspondingly the exiting working fluid becomes closer to the cold target temperature, T_C. Therefore the temperature difference between the exiting working fluid and cold target temperature (\(T_{fl}^{(N)} - T_c\)) decreases and regenerator effectiveness increases, according to Eq. (23). Fig. 6 also reveals that regenerators composed of multiple sub-regenerators maintain a linear temperature distribution, which is crucial for enhancing the regenerator effectiveness. This linear temperature profile also indicates that average regenerator temperatures should be evaluated using an arithmetic mean between the first and last sub-regenerators, rather than using a log-mean temperature difference, as is often done in the literature. In summary, increasing the number of sub-regenerators enables the last sub-regenerator and exiting working fluid to achieve a temperature closer to the target temperature, which enhances the regenerator’s effectiveness. We will now quantify how the number of sub-regenerators and regenerator thermal mass ratio influence the effectiveness of Stirling engine regenerators.

4.2.2. Multiple sub-regenerator effectiveness

The regenerator effectiveness from Eq. (25) is plotted against the number of sub-regenerators with various thermal mass ratio values in Fig. 7, which shows that increasing both the regenerator thermal mass ratio and number of sub-regenerators enhances the effectiveness of the regenerator. To determine the maximum attainable effectiveness, an infinite regenerator thermal mass ratio (\( γ = \infty \)) is substituted into Eq. (25) to yield:

\[
\varepsilon_{reg,max} = \frac{N}{N + 1}
\]

which represents the maximum effectiveness that a regenerator can attain based on the number of sub-regenerators employed (N), analogous to the Carnot efficiency for heat engines. The maximum effectiveness from Eq. (33) is plotted in Fig. 7 to show the upper limitation on the regenerator effectiveness value versus the number of sub-regenerators used.

We can also determine how to achieve a specific value for regenerator effectiveness based on the number of sub-regenerators (N) and regenerator thermal mass ratio (\( γ \)) used in the design of a Stirling engine regenerator. Fig. 8 is a plot of Eq. (25), which shows how to select the values of both the number of sub-regenerators and the...
regenerator thermal mass ratio to achieve regenerator effectiveness values of 95%, 96%, 97%, and 98%. Any values above and to the right of the effectiveness lines will result in a higher regenerator effectiveness. It is also shown in Fig. 8 that the effectiveness lines approach a vertical asymptote as the number of sub-regenerators decreases. This occurs since each regenerator effectiveness has a corresponding minimum required number of sub-regenerators to attain the specified effectiveness. For example, to attain 95% regenerator effectiveness, the minimum required number of sub-regenerators is 19, and it is impossible for a regenerator with less than 19 sub-regenerators to attain 95% effectiveness. Stirling engine designers can use Eq. (25) to select a regenerator effectiveness they desire based on the selected regenerator thermal mass ratio and number of sub-regenerators employed.

4.2.3. Influence of mass distribution

An interesting question that arises in the design of regenerators is: does the distribution of the sub-regenerator mass influence the regenerator effectiveness? To simulate an uneven mass distribution, the regenerator thermal mass ratio values in Eqs. (16)–(19) were varied for each of the sub-regenerators over a wide range of distributions. The analysis revealed that the regenerator effectiveness is a function of the non-uniformity of the mass distribution, which is described by the standard deviation of the regenerator thermal mass ratio, σ, and the arrangement of the distribution did not matter. The regenerator effectiveness is plotted against the standard deviation in Fig. 9, which shows that as the standard deviation increases, which corresponds to an
increasingly uneven distribution of the sub-regenerator mass, the regenerator effectiveness decreases until it reaches the value for a single regenerator (with the corresponding regenerator thermal mass ratio). The reason that the regenerator effectiveness approaches the value for a single regenerator as the standard deviation increases is because an extremely uneven mass distribution approximates that of a single regenerator. It is concluded from this analysis that the optimum sub-regenerator mass distribution for a Stirling engine regenerator is a uniform mass distribution.

4.2.4. Experimental validation

Validation of the heat transfer model is necessary to ascertain that the derived expressions can generate accurate predictions of the thermal behaviour in Stirling engine regenerators. To ensure that the experiments were eligible to be compared against the heat transfer model developed in Section 2.3, the conditions described in Eqs. (3) and (13) had to be satisfied. The highest resultant Biot number from the experiments was $6.5 \times 10^{-4}$, and the highest regeneration time ratio value, $\zeta$, was 0.82, thus demonstrating that the experiments satisfied the conditions. The experimental and theoretical results of regenerative heating and cooling processes for spaced and unspaced sub-regenerator configurations are shown in Fig. 10(a) and (b), for 12 and 16 sub-regenerators, respectively. It is shown that there is good agreement between the experimental and theoretical results, for target temperatures of $T_H = 353 \, \text{K (80 °C)}$ and $T_L = 303 \, \text{K (30 °C)}$, with percent differences ranging between 0.14% to 2.4% for 12 sub-regenerators (0.47 K to 7.73 K), and 0.01% to 1.96% for 16 sub-regenerators (0.03 K to 6.68 K). The experimental results shown in Fig. 10(a) and (b) also confirm that the temperature distribution within the regenerator is linear. Since there is good agreement between the heat transfer model and experimental results, it is confirmed that the heat transfer model has been validated and can be used in the design of Stirling engine regenerators to predict their effectiveness on the selected values of regenerator thermal mass ratio and number of sub-regenerators.

4.2.5. How to thermally isolate sub-regenerators

Another interesting question that arises from the results of this study is: how can a designer ensure that there is sufficient thermal isolation between sub-regenerators to yield a regenerator configuration that has multiple sub-regenerators and is not behaving as a single regenerator? To examine this, we performed experiments with two sub-regenerator configurations: spaced and unspaced, where the wire meshes used to form the regenerator were either directly in contact or separated by a ceramic insulation ring, respectively. It is shown in Fig. 10(a) and (b) that the experimental values of spaced and unspaced configurations are nearly identical, with an average percent difference of 0.53% for 12 sub-regenerators (1.78 K), and 1.21% for 16 sub-regenerators (4.06 K). Fig. 10(a) and (b) reveal that unspaced sub-regenerators, such as wire meshes stacked directly against one another, provide sufficient thermal contact resistance to be considered as thermally isolated for the temperature range used in the experiments ($T_H = 353 \, \text{K (80 °C)}$ and $T_L = 303 \, \text{K (30 °C)}$). Therefore, Stirling engine designers can simply stack wire mesh sub-regenerators together to attain the required thermal resistance to produce a sufficient temperature distribution, rather than needing to space each individual sub-regenerator or use other complex isolation strategies.

4.3. Stirling engine efficiency

The efficiency of Stirling engines is ultimately the most important outcome when designing regenerators. We perform a parametric analysis using Eq. (32), to examine the effects of the regenerator thermal mass ratio and number of sub-regenerators on the efficiency of Stirling engines, and compare the resultant efficiencies with that of the Carnot cycle (ideal regeneration).

4.3.1. Effect of number of sub-regenerators

The number of sub-regenerators directly influences the effectiveness of the regenerator, which subsequently impacts the thermal efficiency of the Stirling engine, as seen in Eq. (32). In this portion of the parametric analysis, the number of sub-regenerators was varied from 1 to 25 for various target temperature ratios, $T_H/T_L$, while the compression ratio was fixed at a value of 10:1, the employed working fluid was hydrogen, and the regenerator thermal mass ratio was fixed at a value of 20:1. It is shown in Fig. 11(a) that as the number of sub-regenerators increases, Stirling engine efficiency also increases, as expected with an increasing regenerator effectiveness. It is seen, however, that the incremental increase in Stirling engine efficiency decreases as the number of sub-regenerators increase, illustrated by the lines becoming progressively closer together, which indicates that there are diminishing returns as the number of sub-regeners is increased. Therefore, Fig. 11(a) can be used by Stirling engine designers to select an appropriate number of sub-regenerators.
appropriate value for the number of sub-regenerators required to achieve a desired value for the Stirling engine efficiency. The selection of an appropriate number of sub-regenerators will avoid the use of an excessive number of sub-regenerators, which could impede the flow of the working fluid through the regenerator and increase the pressure drop.

4.3.2. Effect of regenerator thermal mass ratio

The regenerator thermal mass ratio also influences the effectiveness of the regenerator, which subsequently impacts the thermal efficiency of the Stirling engine, as seen in Eq. (32). In this portion of the parametric analysis, the value of the regenerator thermal mass ratio was varied from 1:1 to 10:1 for various target temperature ratios, $T_H/T_L$, while the compression ratio was fixed at a value of 10:1, the employed working fluid was hydrogen, and the number of sub-regenerators was fixed at a value of 25. It is shown in Fig. 11(b) that as the value of the regenerator thermal mass ratio increases, the Stirling engine’s efficiency also increases, as expected with an increasing regenerator effectiveness. There are also diminishing returns as the regenerator thermal mass ratio approaches $\infty$:1, as illustrated by the lines becoming progressively closer together. Similarly to the number of sub-regenerators, Fig. 11(b) can be used by Stirling engine designers to select an appropriate value of the regenerator thermal mass ratio required to achieve a desired value for the Stirling engine efficiency. The selection of an appropriate value of the regenerator thermal mass ratio will avoid the use of an unnecessary amount of regenerator mass.

5. Conclusions

In this work, a discrete one-dimensional heat transfer model was developed to determine which parameters influence the effectiveness of Stirling engine regenerators. The heat transfer model revealed that the regenerator thermal mass ratio and number of sub-regenerators were the two parameters that influence the regenerator effectiveness and quantify how they influence it. The use of multiple sub-regenerators was shown to produce a linear temperature distribution within a regenerator, which enables the effectiveness to be increased above 50%. It was determined that increasing the regenerator thermal mass ratio and number of sub-regenerators resulted in an increase in regenerator effectiveness and a corresponding increase in the Stirling engine efficiency. It was also found that the regenerator effectiveness has a maximum attainable value, which is limited by the number of sub-regenerators used. For example, to attain a regenerator effectiveness of 95%, a minimum of 19 sub-regenerators must be used. It was also determined that a uniform sub-regenerator mass distribution provides the highest regenerator effectiveness. Experiments using a regenerator test apparatus, with a temperature range of $T_H = 353$ K (80 °C) and $T_L = 303$ K (30 °C), validated the heat transfer model, and demonstrated that stacking sub-regenerators, such as wire meshes, provides sufficient thermal resistance to generate a temperature distribution throughout the regenerator, which enhances regenerator effectiveness. As a result of this study, Stirling engine designers can select a value of regenerator thermal mass ratio and number of sub-regenerators to attain a desired regenerator effectiveness and/or a desired value for the Stirling engine efficiency.

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References


